13 **Graphs**
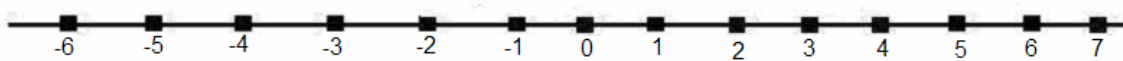
Previous ideas

The number line

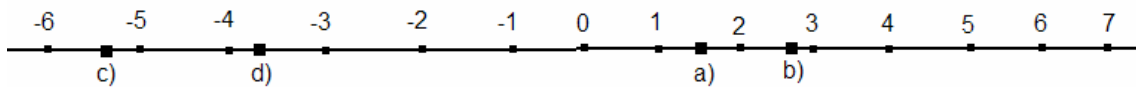Numbers can be represented on a line in this way
1. Draw a line.
2. Choose a point for zero.
3. Positive numbers are drawn to the right, negatives to the left.

Decimal numbers can also be placed on the line

Example 1 Place approximately on the number line the following numbers

a) 1.5   b) 2.8   c) - 5.3   d) - 3.7

1 Graphs

How to plot points on a plane

To represent points in a plane we use two perpendicular number lines.

The horizontal line is called the x-axis (positives numbers to the right and negatives to the left).

The vertical line is called the y-axis (positive numbers up and negatives down).
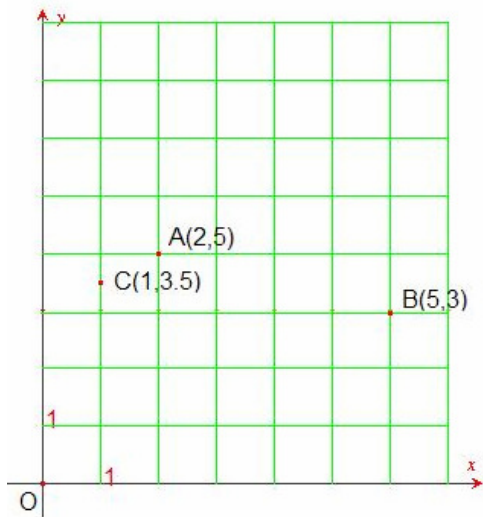
The common point of the two lines is called the origin O

For plotting a point we need an **ordered** pair of numbers, be careful! The order in which the couple is written is important.
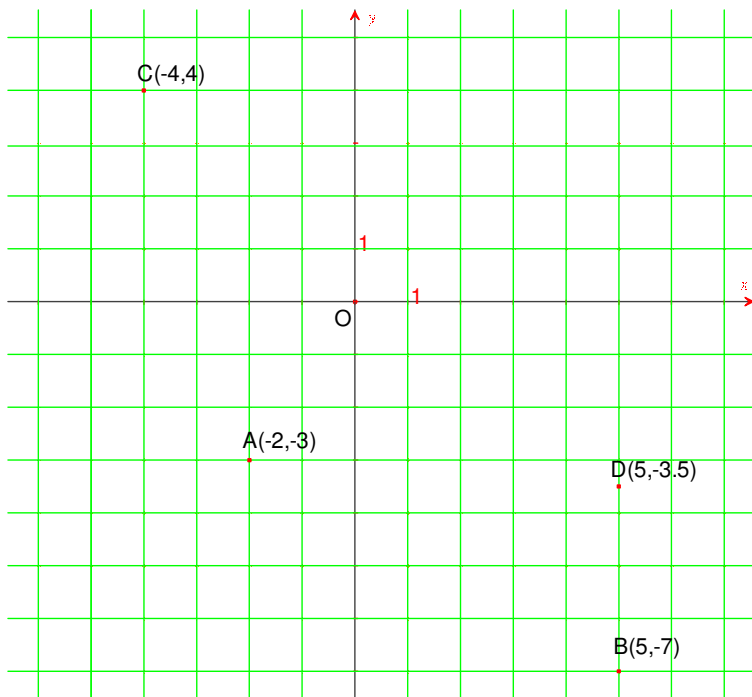
The firs number of the pair is the x-coordinate (abscissa).

The second one is the y-coordinate (ordinate).

Example 2 Plot the numbers A(2,5) B(5,3) and C(1,3.5)

*Example 3* Plot the numbers A(-2,-3) B(5,-7) and C(-4,4) and D(5,-3.5)

Exercises

Exercise 1 In the graph data is plotted about the height and weight of a group of students, where x is the height in cm and y is the weight in kg of each one. Answer these questions.

   a) Who is the tallest and how tall is she/he.

   b) Who is the heaviest and what is his/her weight.

   c) How much heavier is Antonio than Alex?
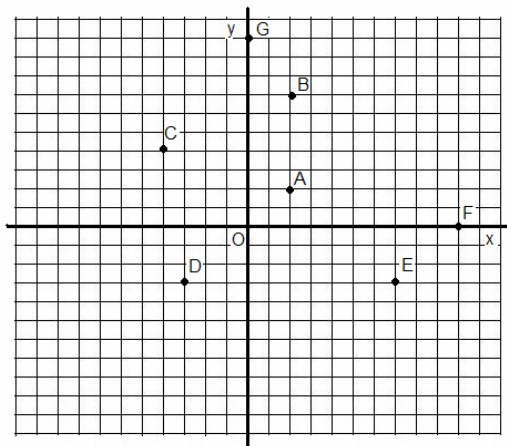
d) Who are taller than Vanessa and who are shorter?

e) In this case, is it true that the taller people are generally heavier?
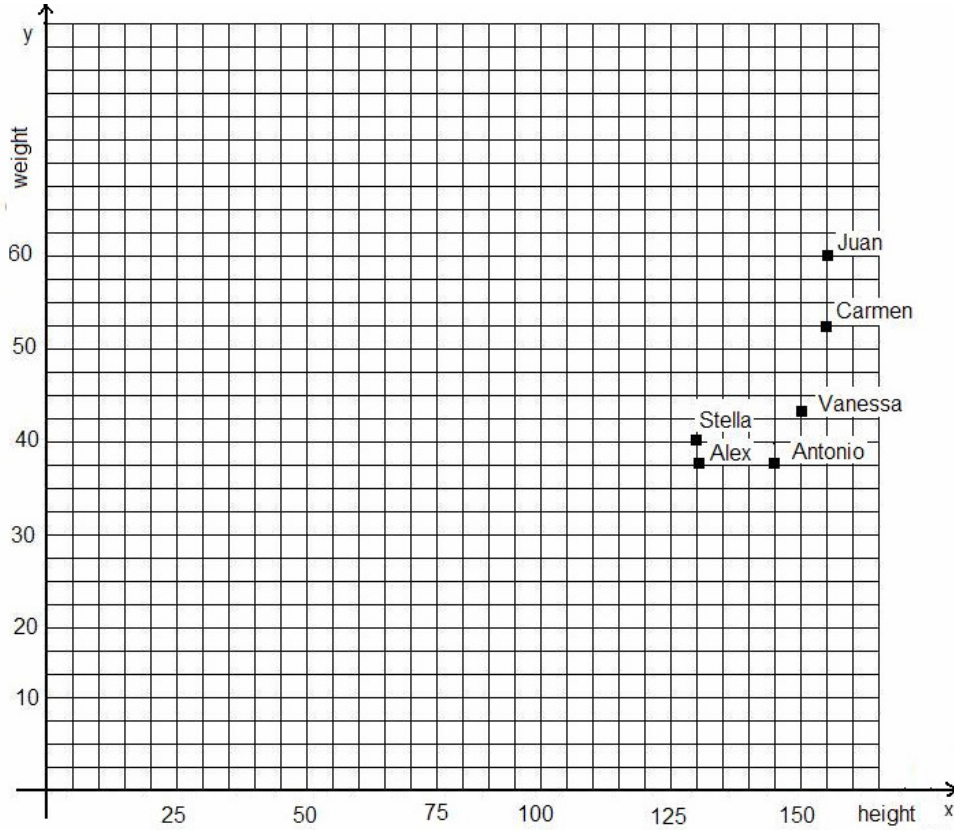
This is a scatter graph

Exercise 2

a) Write the coordinates of A, B, C, D, E, F, and G

b) Which two points have the same x-coordinate?

c) Which two points have the same y-coordinate?

d) Which points have the same x and y coordinates?

e) Plot a point P which has the same x coordinate as C and the same y coordinate as A.

f) Plot a point, which has the same coordinates as B but in the opposite order.

Exercise 3

a) Plot the points A(-2,-2); B(0,0); C(4,4) and D(5.5,5.5).

b) Draw the line that joins all of them.

c) Plot P(5,7) and P'(7,5). What can you see?

d) Plot Q(-2,4) and Q'(4,-2). What can you see?

The points P' and Q' are the images of the points P and Q under reflection of the symmetric line drawn in b).

e) Plot the points R(-4,-1) and the symmetric R'

f) Plot the points S(5,2) and the symmetric S'

Exercise 4 Draw a grid. Join these points in the order they appear.

1. (2,2); (2,10); (9,10); (9,2); (2,2)
2. (3,7); (3,8); (4,8); (4,7); (3,7)
3. (7,7); (7,8); (8,8); ((8,7); (7,7)
4. (5,5); (5,7); (6,7); (6,5); (5,5)
5. (4,3); (4,4); (7,4); (7,3); (4,3)

Exercise 5 Write a set of instructions to a friend to see if she/he can write the school initials displayed on the grid.
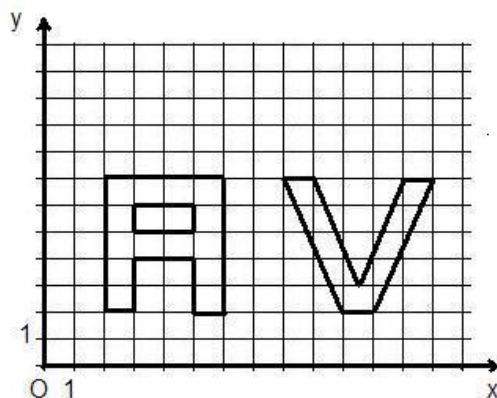
Exercise 6 Write the coordinates of the points that are plotted on the Cartesian diagram.
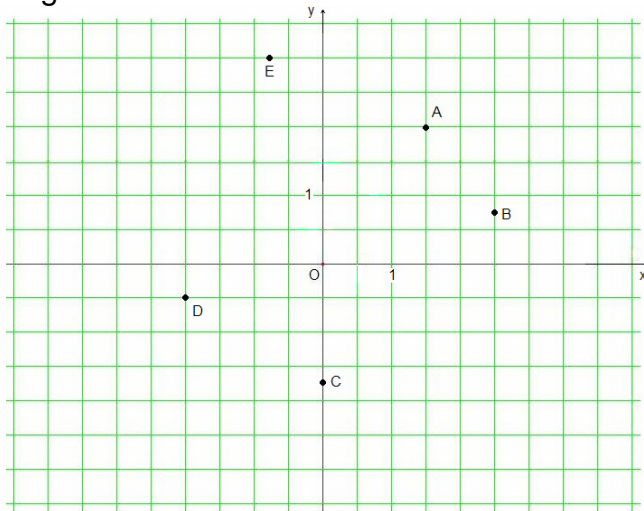
**Exercise 7** For each plotting draw a new coordinate diagram

7.1 Plot the points, join them in order and name the shapes

   a) (2,2); (2,5); (6,5); (6,2); (2,2)

   b) (2,6); (4,4); (7,7); (2,6)

7.2 Write down the coordinates of the missing point and name the shape.
(1,2); (2,5); (5,4); (,)

7.3 Write down the coordinates of the missing points the shape is ABCDEFGH its opposite angles are the same, but the sides are not all of the same length.
A(0,0); B(-2,2); C(-2,4); D(0,6); E(3,6)...

Exercise 8 Look at the map of this small island and:
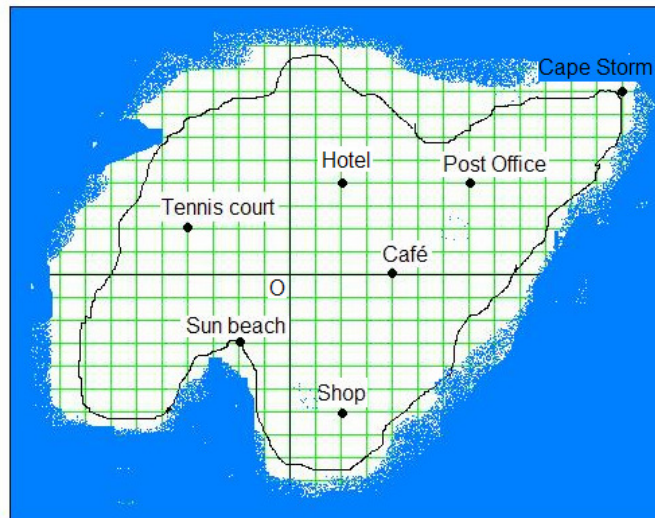

Scale
1 square = 0.5 km

a) Write the coordinates of every important place in the island.

b) Can Peter walk in a straight line from coordinates (-7,-4) to the shop?

c) Tom is on Sun beach, he travels 3.5 km north and 2 km east. Where is he now?

2 Graph of a function

2.1 Definitions.

Graphs describe relationships between two different quantities; from this relationship we can build up a set of pairs and draw a graph.

Example 4 Tomatoes are sold at €1.5 per kg, we can plot a graph showing the cost depending on the number of kg bought like this:
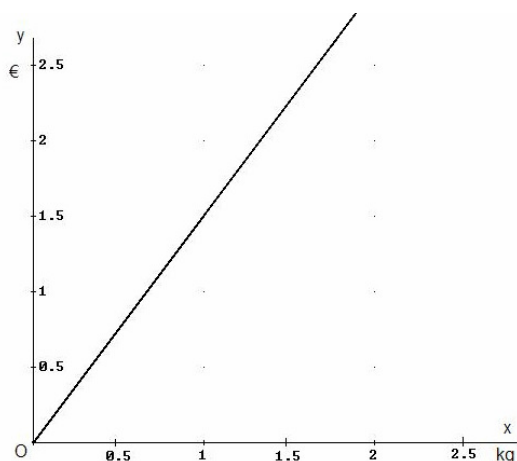
You can see that all the points are on a line

These magnitudes are directly proportional, when one doubles the second will be doubled also.
In general graphs describe the relationship between two variables x and y

x is the independent variable (in the example kg of tomatoes).

y is the dependent variable (in the example price of the purchase).

We see that y depends on x

Notice that for each x-value, there is one and only one possible y-value. This is important!

Sometimes it is easy to describe the relationship by a formula in the example the formula is y = 1.5·x

2.1 Conversion graphs.

They are used to convert between different units of a magnitude.

Example 5 converting miles into km and vice versa. We know that 1 mile is approximately 1.609 km and they are directly proportional. We can draw a graph and use it to convert between these units. It is useful using a drawing triangle.
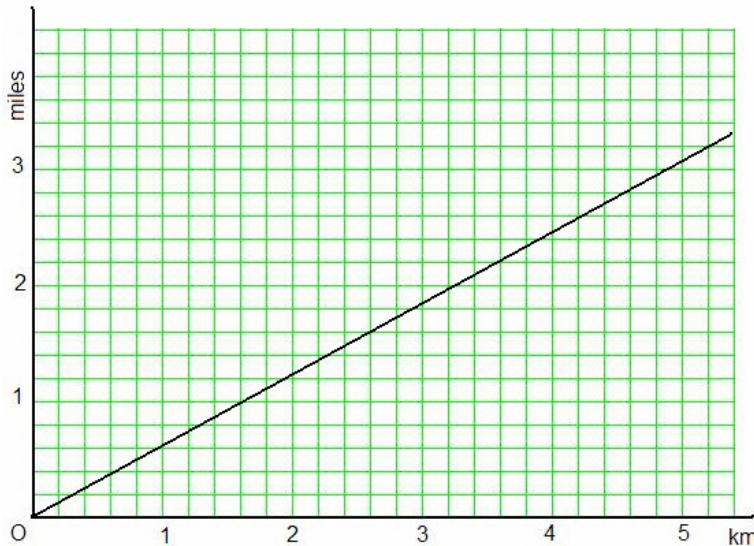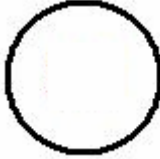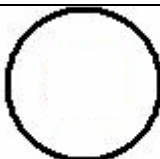

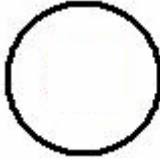From the graph you can see that 2.5 miles is about 4 km, 3.6 km are about 2.2 miles or 3 km are approximately 1.9 miles.
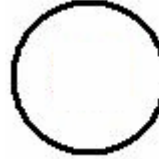
Can you convert three different lengths in km into miles and three more from miles into km?

Exercises

Exercise 9 If $\boxed{50\ mph}$ in U.K. is the same as $\boxed{80}$ km/h in Europe

   a) Draw a line graph for the conversion between mph and km/h

   b) Use the graph to complete the tables:

Speed limit mph In U.K.	Speed limit in km/h In Europe
30 mph	
40 mph	
70 mph	

Speed limit in km/h In Europe	Speed limit mph In U.K.
40	
90	
120	

Exercise 10 Draw a graph to convert € into $ and vice versa. (Use the conversion rate of €10 = 14 $).

Use the graph to convert approximately to the other unit of money:
a) $12   b) €12   c) $20   d) €7   e) $16

Exercise 11 There are two main temperature scales, Fahrenheit which is very common in U.S. and Celsius. These are the data related to the freezing and boiling temperatures of the water.

	Freezing point	Boiling point
Celsius	0ºC	100ºC
Fahrenheit	32ºF	212ºF

a) Draw a graph line through the two points given.

b) Use it to convert these temperatures into the other scale.

150ºF	70ºF	90ºF	100ºF	40ºF
15ºC	25ºC	37ºC	40ºC	60C

Exercise 12 The graph below describes the time required to defrost a piece of meat in a microwave oven depending of the weight.

How long is needed to defrost pieces of meat of:

a) 200g

b) 600g

c) 0.5 kg

d) 300 g

e) 1kg and 200g
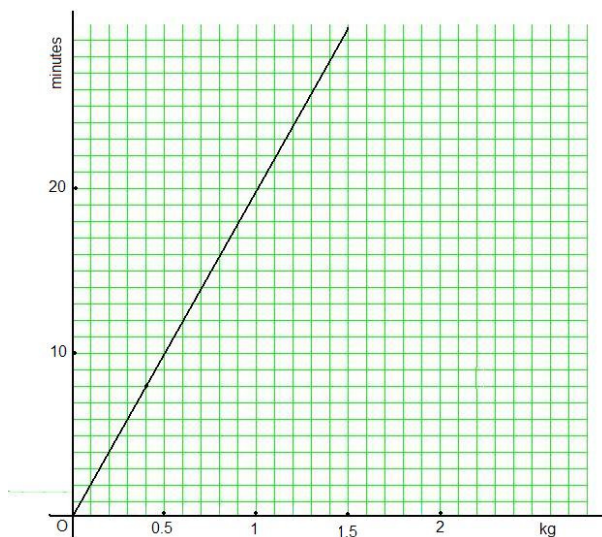
What mass of meat can be defrosted in:

a) 8min

b) 15min

c) 1/2h

How much longer it takes to defrost 300g than 350g of meat?

Exercise 13 This is the height of Wendy recorded every year since she was 4
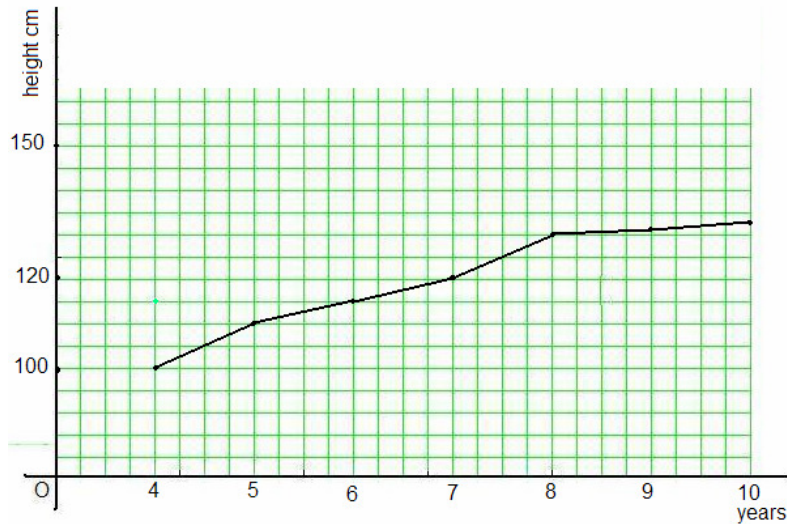
a) How tall was Wendy at 7?
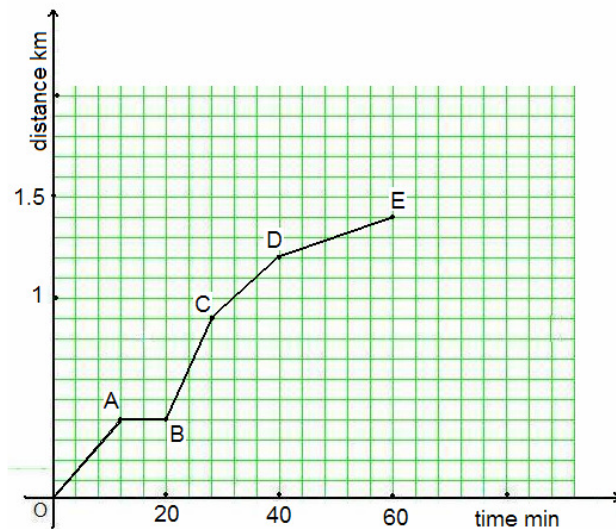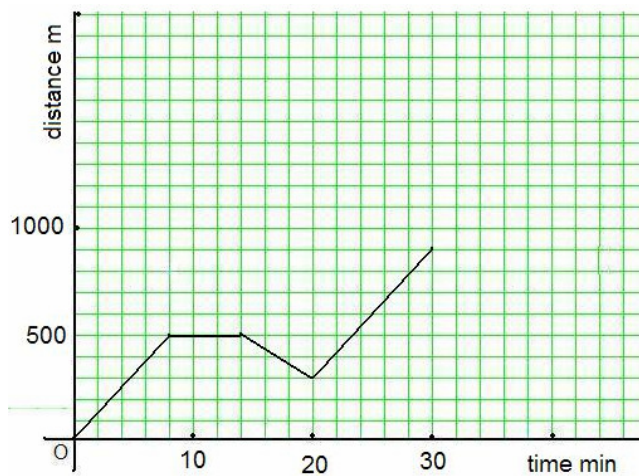
b) How old was she when she reached 132 cm?

c) Between which years she grow faster?

Exercise 14 In this graph we plot the distance travelled by Jane on her way to the school.
Describe the journey.

Exercise 15 The distance from Paul's house to the school is on the graph. Describe his way to school.

3. Handling with data.

When we need to analyse data they must be collected and organized in a table.

Data can be categorical or numerical.

Examples of categorical data are: colour, kind of music, foods, our favourite subject…

Numerical data can be discrete or continuous.

Examples of numerical discrete data are: size of shoes, number of brothers and sisters…

Examples of numerical continuous data are: height, measures of length, …

Exercise 16 Classify as categorical, numerical discrete or numerical continuous the following.

Weight, length of words, favourite sport, number of leaves of a plant, a house number, time practising a sport, favourite TV program, calories of different foods, mark in the last test of maths.

Think of some other types of data on your own and classify them.

3.1 Organising data. Frequency tables

It is advisable to follow these steps.

1. Collect data.

2. Organise data and display them in a frequency table.

3. Draw a graph.

Example 6 30 students have been asked about the number of brothers and sisters they are in their families. These are the answers:

1. Collect data: 1, 2, 1, 3, 6, 3, 2, 1, 1, 1, 2, 2, 3, 2, 3, 2, 2, 4, 2, 3, 3, 2, 2, 3, 4, 2, 2, 3, 1, and 2.

2. Organise data into a frequency table:

Number of B/S	Tally	Frequency
1	IIIII I	6
2	IIIII IIIII III	13
3	IIIII III	8
4	II	2
5		0
6	I	1

Then a bar chart can be drawn with this information.
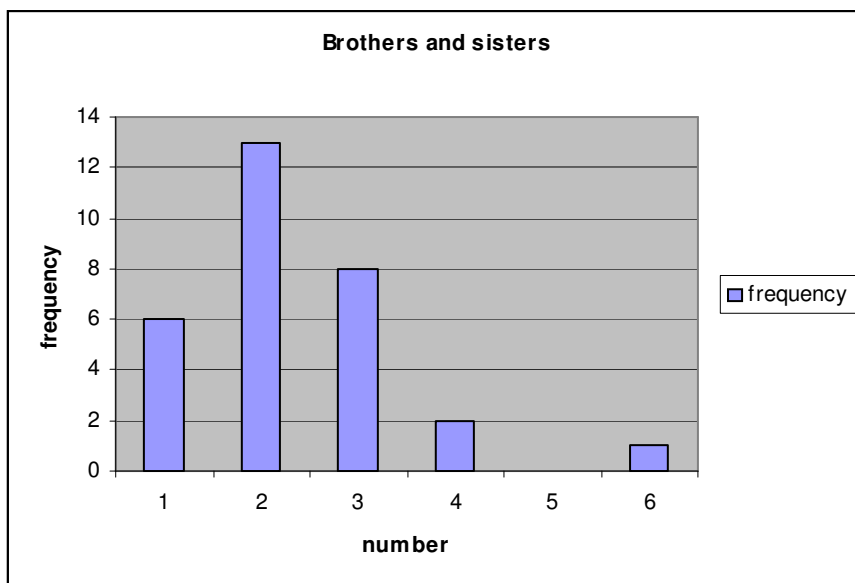
There are other types of graphs for example a pie chart; in this we draw sectors that represent the proportion in each category.

Example 7 Using the same data from the previous example we can organize them in this way:

| Type of family	Frequency
One child	6
Two children	13
Large family	11

If 30 people is 360º, that is the full circle
6 people would be x and:

$x = \dfrac{6 \cdot 360}{30} = 72º$, and repeating for the rest of data we get:

Type of family	Frequency	Degrees
One child	6	72º
Two children	13	156º
Large family	11	132º
Total	30	360º

A formula can be used: $\text{Angle} = \dfrac{\text{frequency} \cdot 360}{\text{Total data}}$
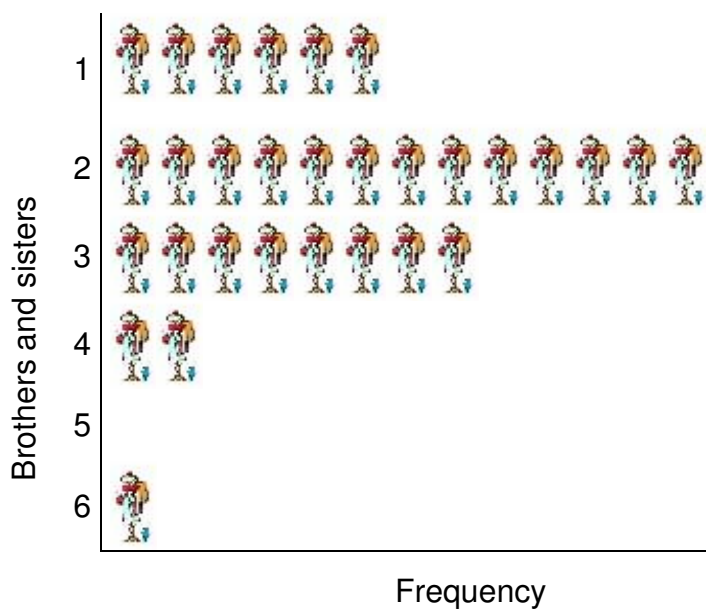
Sectors might be labelled.
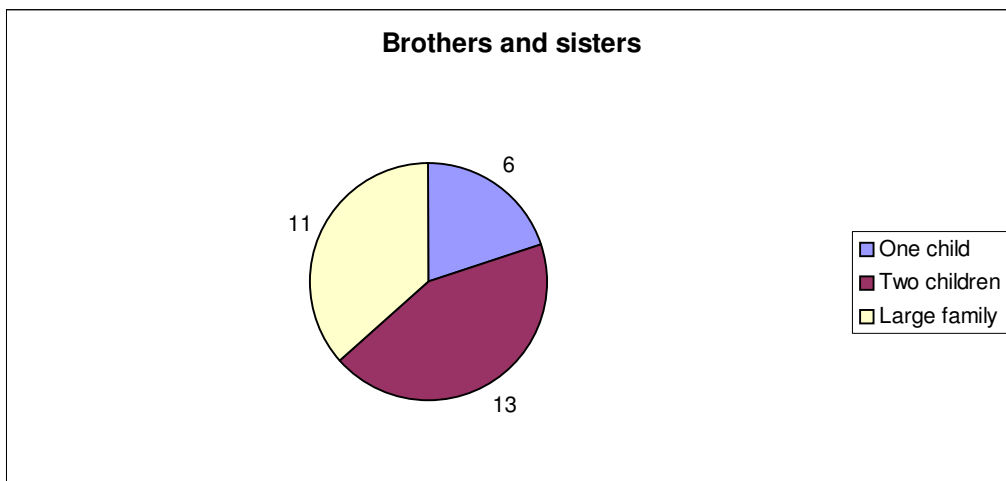
Another type of graph is the pictogram.

Example 8 Using the same data from the example 6 we would have:

Number of brothers and sisters	Frequency
1	6
2	13
3	8
4	2
5	0
6	1

And a pictogram:

Exercises

Exercise 17 Copy and organize in a frequency table the number of vowels in these texts.

- *"Pure mathematics is in its way, the poetry of logical ideas".* **(Einstein).**
- *"The laws of Nature are but the mathematical thoughts of God".* **(Euclid's).**

Draw a frequency chart and a pie chart with the data.

Translate the two sentences into Spanish and do the same as above.

Exercise 18 Roll a dice 30 times and record the scores
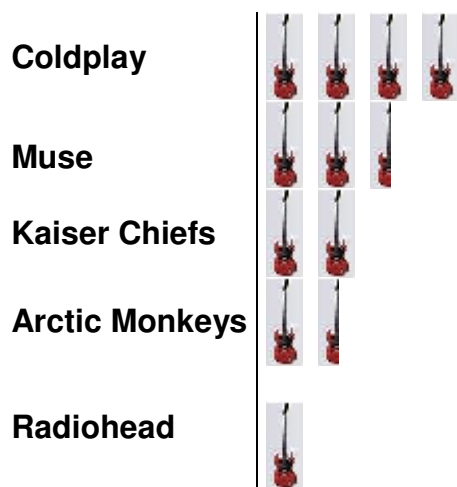
Organize the data in a table

Draw a bar chart

Which is the most frequent score?

Is that what you expected?

How could you improve the reliability of the experiment?

Exercise 19 The results for the best British band survey in a high school are represented in this pictogram

Coldplay

Muse

Kaiser Chiefs

Arctic Monkeys

Radiohead

Where represents 100 votes

Organize the data in a table

Draw a pie chart

Exercise 20 The marks in a test of mathematics have been:
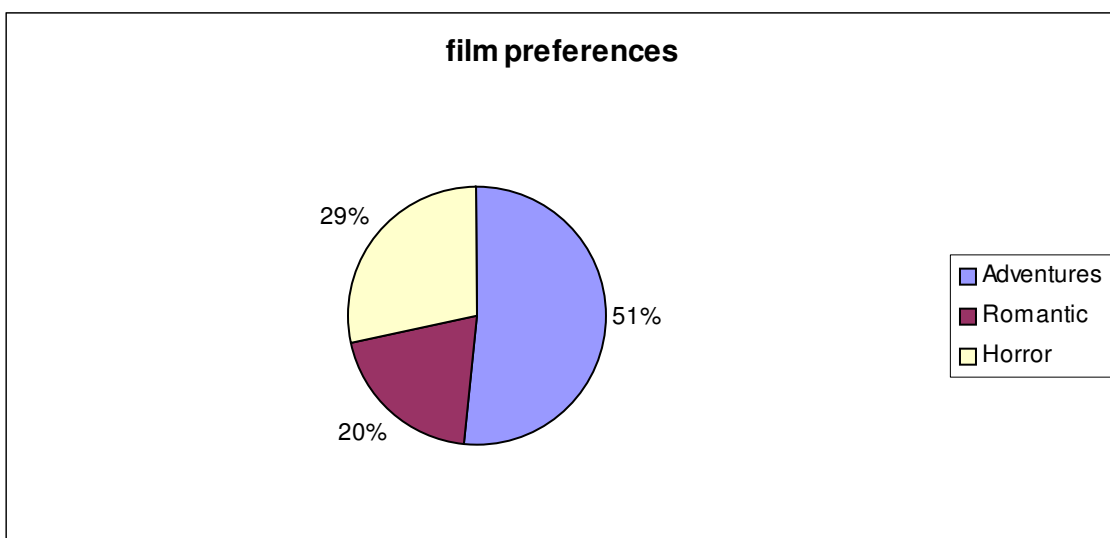
2 3 8 7 7 3 9 5 5 4 1 6 5 7 6 6 5 8 7 3 4 5 6 5 4 6 7 7 9 8

Organize the data in a frequency table

Draw a bar chart

Draw a pie chart organizing the data as IN, SF, B, NT, SB

Exercise 21 In a survey, a group of 80 people were asked which kind of films they liked the most. The answers are represented in this pie chart.

How many people like each kind of film?

Organize the data into a frequency table

Draw a bar chart

Which of the two graphs is the best for describing the data?

Exercise 22 This is the 24-hr average temperature of 118 months between 1981 and 1990 recorded in the Weather station ALBACETE/LOS LLANOS

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year
℃	5.1	6.5	9.5	11.4	15.3	20.9	24.5	23.9	20.8	14.8	9.8	6.3	14.2
℉	41.2	43.7	49.1	52.5	59.5	69.6	76.1	75.0	69.4	58.6	49.6	43.3	57.6

Draw a bar chart with the temperatures in ºC

Draw a line graph with the temperatures in ºF

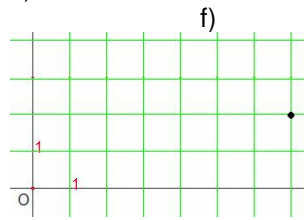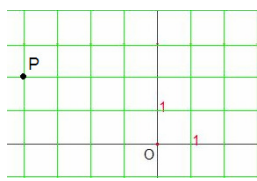**What can be seen in both graphs?**

Solutions

Exercise 1 a) Juan and Carmen, both are 155 cm, b) Juan weights 60 kg, c) Both have the same weight, d) Juan and Carmen are taller than Vanessa, Stella, Alex and Antonio are all shorter, e) No.
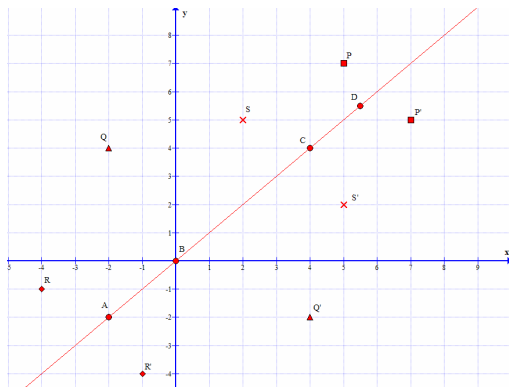
Exercise 2 a) A(2,2); B(2,7); C(-4,4); D(-2,-2); E(7,-3); F(10,0), and G(0,10)
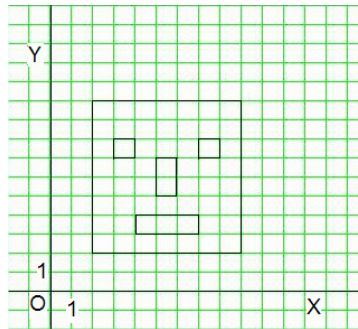b) A and B, c) D and E, d) A and D
e)   f)
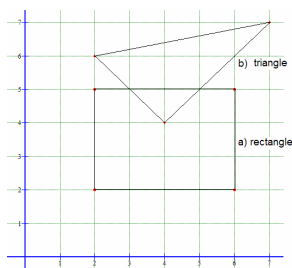

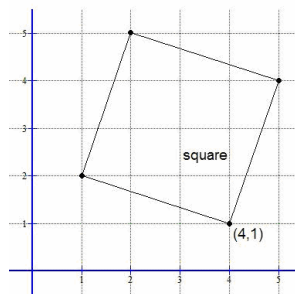**Exercise 3**   **Exercise 4**

Exercise 5 a. (2,2); (2,7); (6,7); (6,2); (5,2); (5,4); (4,4); (4,2); (2,2). b. (3,5); (3,6); (5,6); (5,5); (3,5). c. (8,7); (9,7); (10.5,3); (12,7); (13,7); (11,2); (10,2); (8,7).
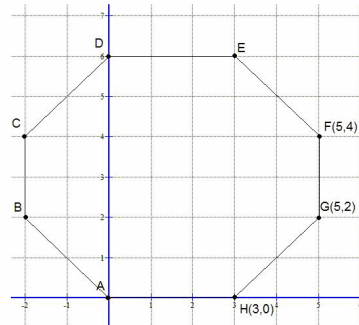**Exercise 6** A(1.5,2); B(2.5,0.75); C(0,1.75); D(-2,-0.5); E(-0.75,3)
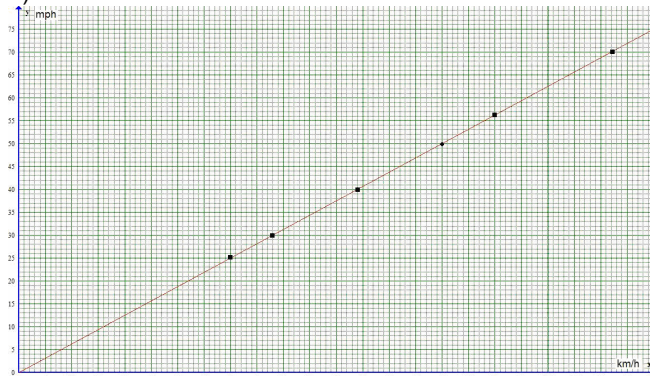**Exercise 7.1**   **7.2**   **7.3**

Exercise 8 a) Cape Storm (13,8)   , Hotel (2,4),   Post Office (7,4), Tennis court (-4,2), Café (4,0), Sun beach (-2,-3), Shop (2,-6); b) No; c) In the hotel.
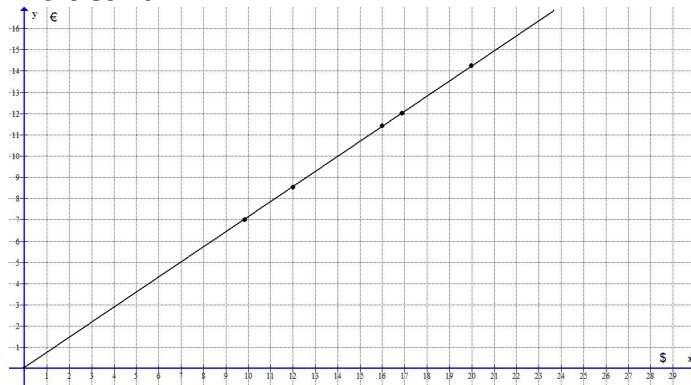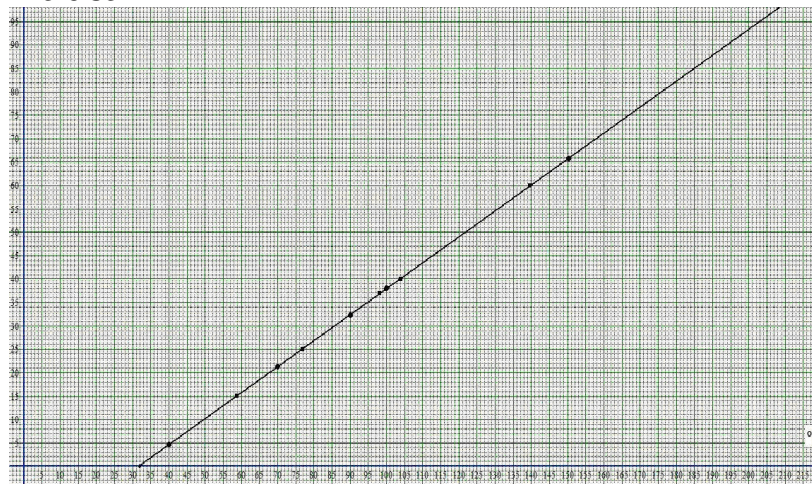
Exercise 9
a)

b) 30 mph = 48 km/h, 40 mph = 64 km/h, 70 mph = 112 km/h,
40 km/h = 25 mph, 90 km/h = 56.5 mph, 120 km/h = 75 mph

Exercise 10

b) \$12 = €8.5 b) €12 = 16.9\$ c) \$20 = €14 d) €7 = \$10 e) \$16 = €11.5

Exercise 11

b) 150ºF= 66ºC, 70ºF = 22ºC, 90º F = 32ºC, 100º F = 37ºC, 40º F = 5ºC 15ºC = 58º F, 25ºC = 73º F, 37ºC = 98º F, 40ºC = 104º F, 60ºC = 140ºF

Exercise 12 a) 200 g = 4 min, b) 600 g = 12 min, c) 0.5 kg = 10 min, d) 300 g = 6 min e) 1 kg and 200 g = 23.5 min

a) 8 min = 40 g, b) 15 min = 750 g, c) 1/2 h = 1.5 kg

1 min more.

Exercise 13 a) 120 cm, b) She was10 years old, c) Between 7 and 8

Exercise 14

O-A She travelled at a constant speed for 12 min and covered 400 m.

A-B She stopped for 8 min.

B-C She ran faster and travelled 500 m in 8 min.

C-D She travelled slowly and got over 300m in 12 min.

D-E She travelled even more slowly and covered the last 200 m in 20 min

In total she travelled 1400 m in 1 hour

Exercise 15

1. He walked for 8 min and got over 500 m

2. He stopped for 6 min or walked in a circle in which the house was the centre.

3. He went back home for 8 min and got 200 m closer to his house.

4. He walked at a constant speed for 10 min and arrived at the school that is 1400 m from his house and lasted 30 min in total.
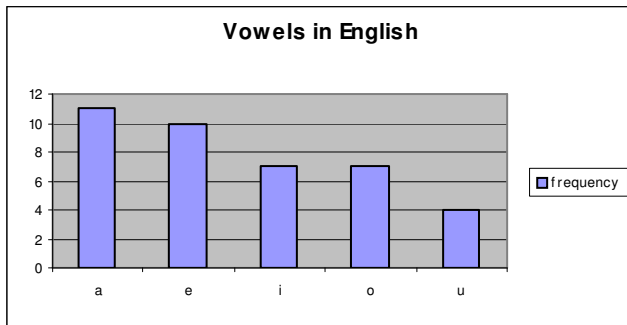
Exercise 16

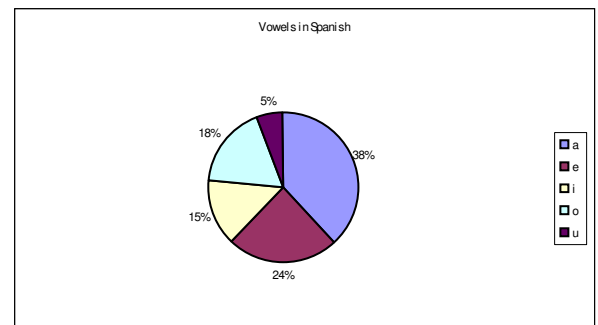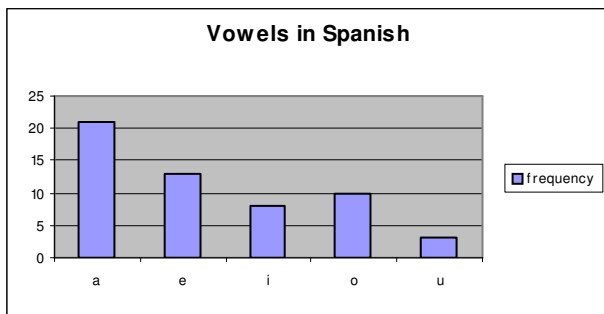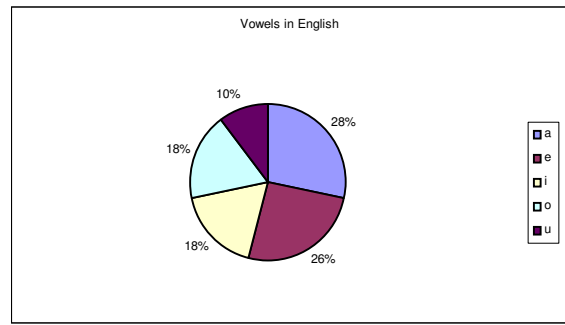| Categorical	Numerical discrete	Numerical continuous
favourite sport	length of words	Weight
Favourite TV program	house's number	number of leaves of a plant
	mark in the last test of maths	time practising a sport
		calories of different foods

Exercise 17

English

Letter	Frequency
a	11
e	10
i	7
o	7
u	4

Spanish

Letter	Frequency
a	21
e	13
i	8
o	10
u	3

Vowels in English

Vowels in English

Vowels in Spanish

Vowels in Spanish

- *"Pure mathematics is in its way, the poetry of logical ideas"*. (Einstein).
- *"The laws of Nature are but the mathematical thoughts of God".* (Euclides).

Translation:
- Las matemáticas puras son a su manera la poesía de las ideas lógicas.
- Las leyes de la Naturaleza no son sino los pensamientos matemáticoss de Dios.
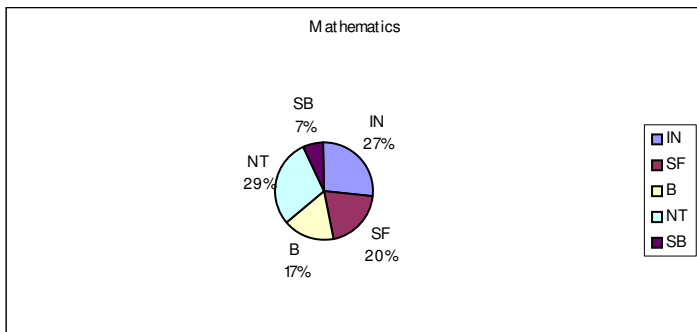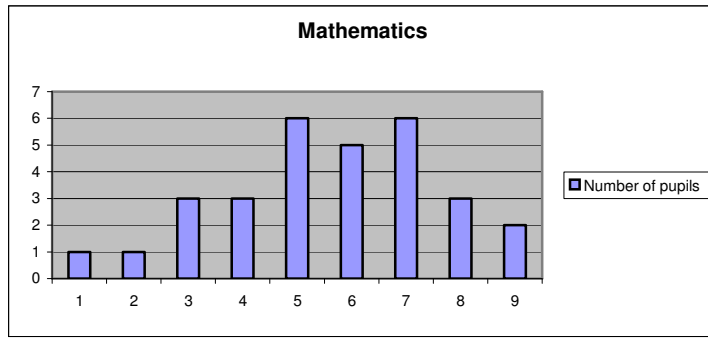
Exercise 18 (It depends on the occurrence)
Exercise 19

Band	Frequency
Coldplay	400
Muse	250
Kaiser Chiefs	200
Arctic Monkeys	150
Radiohead	100

Bands
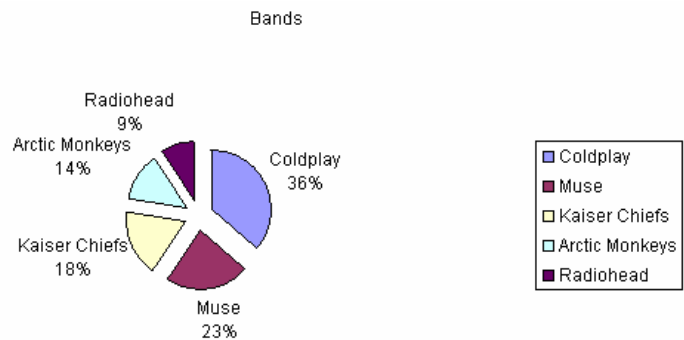
Exercise 20

Marks	Frequency
1	1
2	1
3	3
4	3
5	6
6	5
7	6
8	3
9	2

Mathematics

Exercise 21

Adventures 23   Romantic 41   Horror 16

Film	Frequency
Adventures	23
Romantic	41
Horror	16
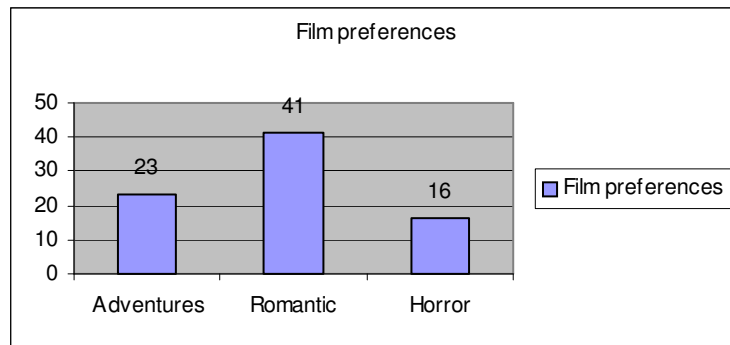
Pie chart is better to compare with the total

Exercise 22
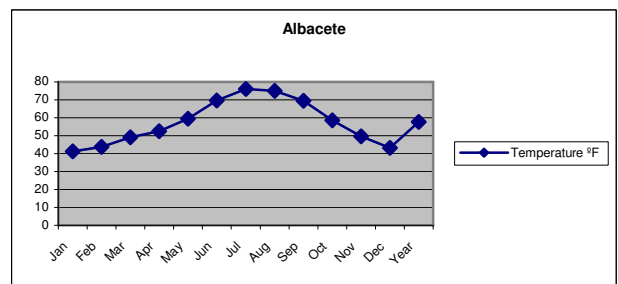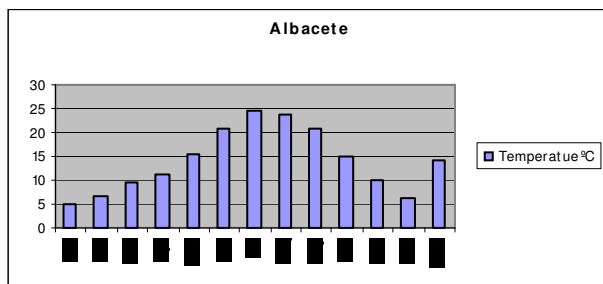
Both graphs are similar because they show exactly the same phenomena.